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Artificial boundary conditions for model updating and damage detection

Gordis, Joshua H.

Monterey, California. Naval Postgraduate School

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Artificial Boundary Conditions for Model Updating and Damage Detection

Joshua H. Gordis
Naval Postgraduate School
Dept. of Mechanical Engineering - Code ME/Go
Monterey, CA 93943-5100
Voice: (408) 656-2866 FAX: (408) 656-2238
Email: gordis@me.nps.navy.mil

ABSTRACT The updating of finite element models commonly makes use of measured modal parameters. The number of modal parameters measured in a typical modal test is small, while the number of model parameters to be adjusted can be large. In this paper, it is shown that the natural frequencies for the structure under a variety of boundary conditions are available from any square, spatially incomplete frequency response function data, without any actual physical alteration of the boundary conditions, hence the term "artificial boundary conditions." This approach can provide potentially a large number of additional and distinct mode frequencies from the same modal test. The simplest example of this approach is that the driving point antiresonance frequencies correspond to the natural frequencies obtained from the structure with the driving point DOF constrained to ground. This result is developed generally for multiple measured points, and is shown to be related to previous results concerning omitted coordinate systems and spatially incomplete test data. The approach is applied to sensitivity-based model updating and damage detection.

NOMENCLATURE

{DV}	Design variables
{f}	Force vector
[H]	FRF matrix
[I]	Identity matrix
[k]	Stiffness matrix
[m]	Mass matrix
[T]	Sensitivity matrix
{x}	Response vector
[Z]	Impedance matrix
a	analysis set (measured coordinates)
ϕ_i^r	i'th element of rth eigenvector
Ω	Forcing frequency (rad/sec)
ω_i	i'th natural frequency (rad/sec)
R	Modal Residue
o	omitted set

1. INTRODUCTION

The improvement, or "updating" of a finite element (FE) model is often a necessary step in order for the model to be used with confidence in the prediction of structural response The inaccuracy in the prediction of dynamic response is gauged most often by the inaccuracy in the prediction of modal parameters. This inaccuracy of the FE model is reduced (the model is "improved") by the adjustment of selected physical and material parameters which define the model. These parameters can include dimensional properties of structural elements, moduli of elasticity, and densities, for example. The selected parameters are adjusted such that the predicted modal parameters calculated from the updated model are brought into closer agreement with the corresponding parameters identified in a modal test, i.e. the measured parameters. The selection of physical parameters to be adjusted is based on a consideration of the accuracy of the parameter values, and from a need to compensate for structural dynamic behavior not accounted for by the finite element formulation.

A typical FE model may be defined by a large number (on the order of 10^2 10^3) of physical parameters. However, a typical modal test of the structure modeled yields a small number (on the order of 10^1) of modal parameters to be used to guide the adjustment of the model parameters. This disparity in the number of known parameters (measured modal parameters) versus the number of parameters to be adjusted defines an underdetermined problem. A further and significant difficulty is that those parameters which are truly in error (if any) are unknown. The solution of this particular problem is known as error localization.

There exists a recognized need to increase the size of the known parameter database. For example, Ref. [1] describes procedures referred to as "Perturbed Boundary Condition (PBC)" testing, where additional configurations of the structure are independently tested. These configurations can include different boundary conditions and addition of mass at selected points on the structure. These procedures require physical modifications to made to the structure, and an additional modal test for each additional configuration.

This paper shows that a large number of additional and distinct mode frequencies can be easily identified from the same modal test performed to identify the standard system mode frequencies, without the need for any physical modification of the structure. These additional and distinct mode frequencies correspond exactly to those mode frequencies found when combinations of measured coordinates are restrained to ground. Hence, these additional frequencies are associated with different boundary conditions for the structure.

The practicality of this result lies in the fact that these boundary conditions need not be actually applied, hence the term "artificial boundary conditions," or "ABC." As a simple example of this, consider the spectrum of antiresonances for any driving point frequency response function. The driving point antiresonance frequencies correspond to the mode frequencies of the structure with the driving point degree-of-freedom (DOF) restrained ("pinned") to ground The ABC's correspond to ideal constraints, i.e. "pins," and therefore can easily be imposed on a FE model. This scheme would yield a separate FE model for each ABC configuration, yet only one set of measured test data. Each FE model is identical except for the boundary conditions, and each model can be used to generate independent sensitivity data, for example.

The ABC are implicitly defined in any set of spatially incomplete frequency response function data, and ark those boundary conditions which define an o-set system or OCS [2,3]. This system is defined by the set of all m-measured coordinates. For a test system, this set is of infinite dimension. ABC configuration frequencies are available from any set of (spatially incomplete) test data due to the fact that a spatially incomplete frequency response function (FRF) matrix is identically equivalent to the FRF matrix which is calculated from the exact dynamic reduction. In short, a measured FRF matrix represents an infinite dimensional system dynamically reduced to the measured coordinates. This is a central fact in this work, and a complete development of this fact is found in [2] and [3].

The OCS has been shown to be central to the effective performance of advanced test-analysis models (TAM's) in model correlation, such as in cross-orthogonality calculations [4]. These background concepts will be briefly developed in what follows.

The artificial boundary condition (ABC) mode frequencies not only provide a greater number of frequencies for the system, but also provides a means to reduce or eliminate ill-conditioning in the solution of sensitivity equations. Since the system is being artificially restrained at various combinations of measured coordinates, the sensitivity matrix columns found from the artificially restrained configuration can be linearly independent from those columns calculated for the baseline configuration of the structure. and it is possible also that a greater number of linearly independent columns can be found from the ABC configuration exclusively, than from the baseline configuration. The use of ABC configurations can allow the discrimination of errors for model parameters which have dependent or nearly dependent sensitivity matrix columns. These *uses* will be demonstrated with simple examples. The reader is referred to Refs. [2-4] for complete expositions of the background theory and its ramifications in model reduction and identification. The theory will be briefly review here to provide a self-contained exposition.

2. THEORY

The equations which define the ABC and OCS will be developed. We first examine fundamental relations in model reduction which define the OCS. We will then develop the analogous relations in the frequency domain which will allow the identification of ABC configuration natural frequencies.

2.1 *OCS and Model Reduction*

We begin with the equation of steady state forced response for a linear structural dynamic system at a forcing frequency Ω (rad/sec):

$$\left[\begin{bmatrix} k_{aa} & k_{ao} \\ k_{oa} & k_{oo} \end{bmatrix} - \Omega^2 \begin{bmatrix} m_{aa} & m_{ao} \\ m_{oa} & m_{oo} \end{bmatrix}\right] \begin{Bmatrix} x_a \\ x_o \end{Bmatrix} = \begin{Bmatrix} f_a \\ f_o \end{Bmatrix} \quad (1a)$$

where k and m are stiffness and mass matrices, x and f are vectors of generalized response and excitation amplitudes, respectively. The subscript "a" refers to the measured coordinates ("analysis set" or "a-set") and the subscript "o" refers to the m-measured coordinates ("omitted set" or "o-set"). Equation (la) can be written as

$$\begin{bmatrix} Z_{aa} & Z_{ao} \\ Z_{oa} & Z_{oo} \end{bmatrix} \begin{Bmatrix} x_a \\ x_o \end{Bmatrix} = \begin{Bmatrix} f_a \\ f_o \end{Bmatrix} \quad (1b)$$

where an impedance matrix is defined as $Z = k - \Omega^2 m$. Assuming no excitations act on the omitted coordinates, the following relationship between the o-set and a-set coordinates is found:

$$\{x_o\} = \left[I - \Omega^2 k_{oo}^{-1} m_{oo}\right]^{-1} \left[-k_{oo}^{-1} k_{oa} + \Omega^2 k_{oo}^{-1} m_{oa}\right] \{x_a\} \quad (2)$$

Equation (2) is the starting point for a class of physical coordinate model reductions [4]. For example, setting $\Omega = 0$ yields the static or Guyan reduction. The derivation of the IRS model reduction transformation starting from Eq. (2) is shown in [4].

The origin of the o-set system is seen from Eq. (2). We replace the bracketed inverse term with its equivalent,

$$\left[I - \Omega^2 k_{oo}^{-1} m_{oo}\right]^{-1} = \frac{\mathrm{Adj}\left[I - \Omega^2 k_{oo}^{-1} m_{oo}\right]}{\mathrm{Det}\left[I - \Omega^2 k_{oo}^{-1} m_{oo}\right]} \quad (3)$$

where Det[•] indicates the determinant and Adj[•] indicates the adjoint matrix. From Eq. (3), we see that the bracketed inverse term, and hence the exact relationship between the a-set and o-set coordinates does not exist at those frequencies Ω which satisfy the equation,

$$\mathrm{Det}\left[I - \Omega^2 k_{oo}^{-1} m_{oo}\right] = 0 \quad (4)$$

The frequencies which satisfy Eq. (4) are the eigenvalues of the system defined by k_{oo} and m_{oo}, i.e. the o-set system, This system is obtained by fully constraining to ground all coordinates in the a-set. As shown in Ref. [4], Equation (2) is the general starting point for the derivation of physical coordinate model reduction transformations, and the existence of the inverse in Eq. (2) above, or the convergence of the series used to replace this inverse, is dependent on the forcing frequency Ω, or the placement of the o-set system frequencies with respect to the system frequencies.

2.2 OCS and Frequency Response Function Matrices

As has been shown [5], the process of instrumenting a structure with a finite number of response transducers defines a reduced order model, where the impedance of the reduced order model is nonlinearly dependent on the impedance of the full order model. Following [5], if we consider that the exact full-order FRF model of a structure is a FRF matrix of infinite dimension (the superscript "x" indicates an experimental quantity),

$$H^x = \begin{bmatrix} H_{aa} & H_{ao} \\ H_{oa} & H_{oo} \end{bmatrix} \tag{5}$$

where the number of coordinates in the o-set is infinite, then the FRF matrix measured in a test is seen to be a matrix partition which has been extracted from the infinite dimension matrix, i.e.

$$\overline{H}^x = H_{aa} \tag{6}$$

where the overbar in Eq. (6) indicates a reduced model. That this measured matrix partition represents a dynamically reduced model is seen as follows. From the partitioned identity ZH=I, we find

$$H_{aa} = \left(Z_{aa} - Z_{ao}Z_{oo}^{-1}Z_{oa}\right)^{-1} \tag{7}$$

The reduced-order impedance relation obtained using the exact dynamic reduction is [3]

$$\{f_a\} = \left[Z_{aa} - Z_{ao}Z_{oo}^{-1}Z_{oa}\right]\{x_a\} \tag{8}$$

The common term in Eqs. (7) and (8) makes clear that a spatially incomplete FRF matrix represents a dynamically reduced model.

The presence of the OCS in a spatially incomplete FRF matrix is contained in the quantity Z_{oo}^{-1} in Eq. (7) The formula for the matrix inverse given by Eq. (3) applies here as well, and since every element in Z_{oo}^{-1} is singular at the natural frequencies of the OCS, we see from Eq. (7) that elements of H_{aa}^{-1} will be singular (or "large, " for a damped system) at the OCS natural frequencies.

3. ABC CONFIGURATION FREQUENCIES: EXAMPLES

The identification of the ABC natural frequencies from measured FRF data using Eq. (7) will he demonstrated with simple numerical examples. The first example, however, will show that driving point antiresonances correspond to the natural frequencies of the structure with the driving point DOF constrained to ground.

Example 1: Driving Point Antiresonances Are ABC Frequencies

We will demonstrate this first using basic principles. Consider the 2-DOF system shown in Fig. 1.

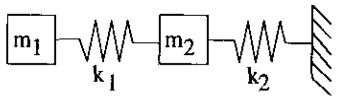

Figure 1. A 2-DOF system

A driving point FRF is given by

$$H_{ii}(\Omega) = \sum_{r=1}^{p} \frac{\left(\psi_i^r\right)^2}{\omega_r^2 - \Omega^2} \tag{9}$$

where ψ_i^r is a mass normalized mode shape element , ω_r is the r^{th} natural frequency, and Ω is the forcing frequency. The frequency of the anti-resonance of $H_{11}(\Omega)$ is given by

$$\Omega_{anti-res}^2 = \frac{R_{11}^1\omega_2^2 + R_{11}^2\omega_1^2}{R_{11}^1 + R_{11}^2} \tag{10}$$

where the modal residue is given by $R_{ij}^r = \phi_i^r\phi_j^r$. It is easily demonstrated that this frequency is equal to the natural frequency of the system in Fig. 1, with the driving point DOF constrained to ground, i.e.

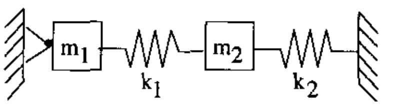

Figure 2. The 2-DOF system with DOF #1 restrained to ground.

If we take $m_1 = m_2 = 1.0$ and $k_1 = k_2 = 1.0$ for the system shown in Fig. I, the frequency of the single antiresonance is $\Omega_{anti-res} = \sqrt{2}$ rad/sec, which equals the single natural frequency of the ABC system, (Fig. 2), which is $\omega = \sqrt{2}$ rad/sec.

Example 2: Calculation of ABC Frequencies for 2-DOF System

The **calculation** of ABC frequencies is done using Eq. (7). A spatially incomplete FRF matrix ***is*** generated (either experimentally or analytically), and this FRF matrix (or submatrices thereof) is inverted at each frequency. The elements of the resulting impedance matrix are plotted versus frequency, and the singular frequencies are the ABC system frequencies. This process can he repeated for various submatrices of the original FRF matrix, thereby generating multiple sets of independent ABC system frequencies. We will repeat Example 1, using the general approach.

We first calculate the driving point FRF $H_{11}(\Omega)$ for the system shown in Fig. 1. Tbis is plotted in Fig. 3. The system has two modes at ω_1=0.6180 radlsec and ω_2=1.6180 radlsec. Using Eq. (10), the anti-resonance frequency is $\Omega_{anti\text{-}res} = \sqrt{2}$ radlsec.

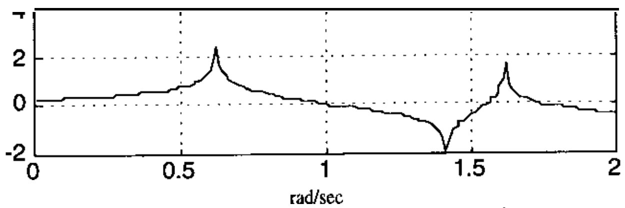

Figure 3. $H_{11}(\Omega)$ versus Ω with $\Omega_{anti\text{-}res} = \sqrt{2}$ (rad/sec)

We now identify the ABC system frequency by calculating $H_{aa}^{-1}(\Omega)$, as per Eq. (7). The a-set contains the single DOF #1 (Fig. 1), and $H_{aa}^{-1}(\Omega)$ is simply the scalar inverse of $H_{11}(\Omega)$. $H_{11}^{-1}(\Omega)$ is plotted in Fig. 4 where the single singular frequency is evident, and directly corresponds to the anti-resonance frequency for this scalar case.

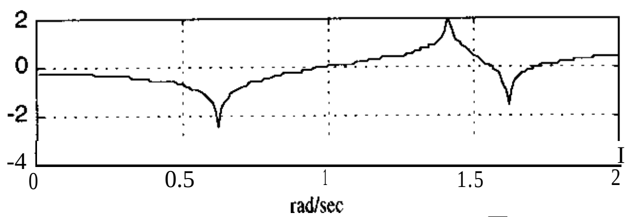

Figure 4. $H_{11}(\Omega)$ versus Ω with $\Omega_{anti\text{-}res} = \sqrt{2}$ (rad/sec)

The singular frequency in $H_{11}^{-1}(\Omega)$ corresponds to the natural frequency of the ABC system obtained by constraining DOF #I (the single measured DOF) to ground. Note that this one-dimensional example demonstrates that the general approach given by Eq. (7), for a single measured coordinate, reduces to the fact that the driving point anti-resonances correspond to the natural frequencies of the system with the driving point coordinate constrained to ground.

Example 2: Calculation of ABC Frequencies for Free-Free Beam

To demonstrate the calculated of ABC system frequencies consider the free-free beam shown in Figure 5, modeled with 10 2-node beam elements. The beam properties are taken as length = 60 inches, El = 5.5E5 lbf-in*, ρ = 0.283 lbf/in^3, and cross-sectional area = 0.75 in^2.

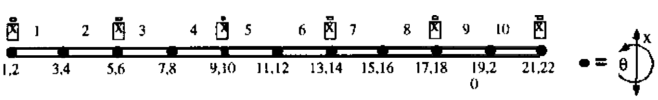

Fig 5. Free-free beam with spatially incomplete transducer set

Accelerometers are shown at DOF's # 1,5,9,13,17, and 21, which are translational coordinates. The a-set is therefore [I 5 9 13 17 21]. We will assume that excitation has been applied at each DOF in the a-set, and hence the FRF matrix is a 6 by 6 matrix. The impedance matrix $H_{aa}^{-1}(\Omega)$ is calculated over a frequency range of O-800 Hr. The driving point FRF, $H_{11}(\Omega)$, is shown in Fig. 6.

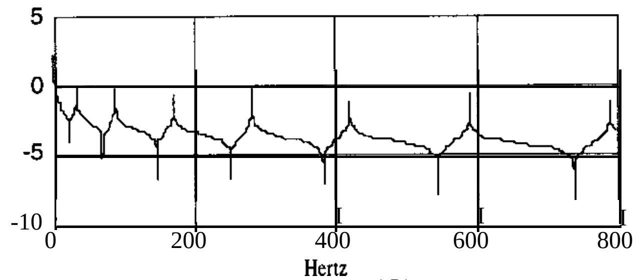

Figure 6. Driving Point FRF, $H_{11}(\Omega)$ for Free-Free Beam

We now identify the ABC system frequencies by calculating the impedance matrix $H_{aa}^{-1}(\Omega)$. If we plot the 1,1 element of this matrix (Fig. 7), the ABC system frequencies are evident:

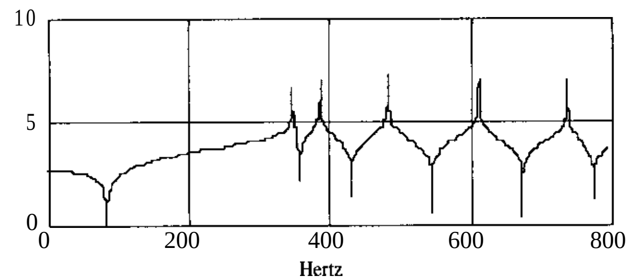

Figure 7. ABC frequencies are the peaks of elements of $H_{aa}^{-1}(\Omega)$.

These frequencies correspond exactly to the natural frequencies of the system obtained when all measured coordinates are constrained to ground, as shown in Fig. 8.

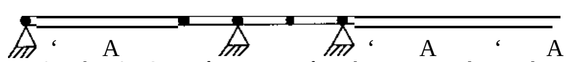

Figure 8. The ABC configuration for the measured coordinates.

As a check, the natural frequencies of the free-free beam, with six pin restraints at the a-set DOF are calculated. These natural frequencies are 346.5, 384.8, 482.4, 610.0, and 735.0 Hertz, which correspond to the peak frequencies in Fig. 7.

Additional sets of ABC configuration frequencies are available from submatrices of H,(R). For example, if we take as a-set DOF's 1 and 17, corresponding to a more common Z-shaker test, the 1,1 element of $H_{aa}^{-1}(\Omega)$ yields an additional set of ABC configuration frequencies, as shown in Fig. 9. The corresponding ABC configuration system is shown in Fig. 10.

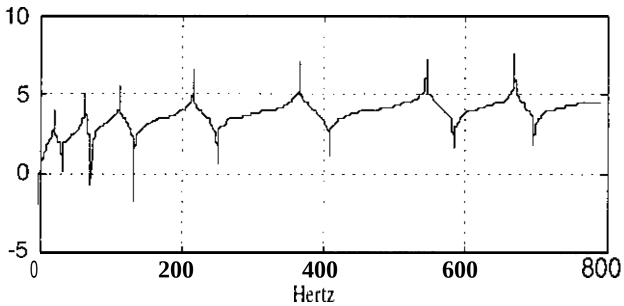

Figure 9. ABC frequencies are peaks of elements of $H_{aa}^{-1}(\Omega)$.

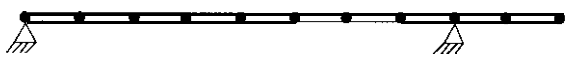

Figure IO. The ABC configuration for the a-set [I 17].

As a check, the natural frequencies for the system shown in Fig. 10 are calculated, and are, 20.45. 63.57. I 12.30. 215.05. 364.60. 543.96, 668.91. 846.93 Hz, which correspond to the peak frequencies in Fig. 9.

4 ABC CONFIGURATION FREQUENCIES IN SENSITIVITY-BASED UPDATING

The ABC configuration frequencies can be used in addition to, or instead of, the standard system mode irequencies in sensitivity-based model updating and damage detection. This is because the ABC frequencies correspond to the same structural system, but with different boundary conditions, The governing equation for sensitivity-based updating is

$$\{\Delta\omega\} = [T]\{\Delta DV\} \tag{11}$$

where $\{\Delta\omega\}$ is a vector of natural frequency errors, { ADVJ is the vector of changes to be calculated for specified model parameters, or "design variable ," and [T] is the matrix of first-order sensitivities, where $T_{ij} = \partial\omega_i/\partial DV_j$. Each ABC system defines additional rows of Eq. (I I). That is, each ABC system defines the following equation:

$$\{\Delta\omega^i\} = [T^i]\{\Delta DV\} \tag{12}$$

where $T_{ij}^k = \partial\omega_i^k/\partial DV_j$ and ω_i^k is the i^{th} natural frequency of the k^{th} ABC configuration system. Quantities associated with the baseline (non-ABC) system will be denoted with the superscript "0". By identifying the irequencies for a total of "K" ABC systems, the total system of equations can be compiled,

$$\begin{Bmatrix} \{\Delta\omega^0\} \\ \{\Delta\omega^1\} \\ \vdots \\ \{\Delta\omega^K\} \end{Bmatrix} = \begin{bmatrix} [T^0] \\ [T^1] \\ \vdots \\ [T^K] \end{bmatrix} \{\Delta DV\} \tag{13}$$

Note that the degree of coupling between the ABC systems and the baseline system in Eq. (13) can be adjusted by deleting or retaining individual columns of the $[T^K]$. For example, consider a system with one ABC configuration. Partial coupling between the baseline system and the ABC system is established if the design variables (DV's) are partitioned such that some of the DV's, say $\{\Delta DV^0\}$, are associated exclusively with the baseline system, some, $\{\Delta DV^{0,1}\}$ with both the baseline and the ABC system. and some exclusively with the ABC system, $\{\Delta DV^1\}$, i.e.,

$$\begin{Bmatrix} \{\Delta\omega^0\} \\ \{\Delta\omega^1\} \end{Bmatrix} = \begin{bmatrix} [T^{0,0}] & [T^{0,1}] & [0] \\ [0] & [T^{1,0}] & [T^{1,1}] \end{bmatrix} \begin{Bmatrix} \Delta DV^0 \\ \Delta DV^{0,1} \\ \Delta DV^1 \end{Bmatrix} \tag{14}$$

where $\{\Delta DV\}$ is so partitioned, and the partitions of the $[T^k]$ are superscripted to emphasize this partial coupling of the equations for the two systems.

The use of ABC system sensitivities can eliminate or reduce the problem of poorly-conditioned or rank deficient [T] matrices. For example, columns of ['I'] can be replaced with columns of $[T^k]$ in order to improve the conditioning. A related problem occurs, for example, when trying to localize damage. Two closely spaced elements in a model will have nearly dependent columns of $[T^0]$, preventing the discrimination of error or damage between the two elements. One column of $[T^0]$ associated with one of the two elements can be replaced with a column of $[T^k]$, and the associated baseline system natural frequency replaced with the ABC system frequency. These applications will be demonstrated as follows.

Example 3: ABC Sensitivity Matrices Can Have Improved Conditioning

Consider again the beam of Example 2 (Fig. 5), in the context of structural damage detection. The sensitivity matrix [T] is generated for the undamaged structure model, in order to solve for the changes in the element EI values representing damage, i.e.

$$\{\Delta\omega^0\} = [T^0]\{\Delta EI\} \tag{15}$$

For the purpose of this example, sensitivities will be calculated for all IO elements, and for the first 10 modes, yielding a square $[T^0]$ matrix of size IO by 10. However, a calculation of rank for this matrix indicates that it is rank deficient,

$$\mathrm{Rank}(T^0)=5$$

and therefore will not provide a fully determined solution for the vector $\{\Delta EI\}$. If excitation has been applied at DOF's 1 and 17 (yielding a square FRF matrix), then it is possible to identify the ABC system frequencies with these DOF constrained to ground. Note that this choice of ABC configuration removes all symmetry from the system boundary conditions. The corresponding ABC system sensitivity matrix is generated, which is also of size IO by IO,

$$\{\Delta\omega^1\} = [T^1]\{\Delta EI\} \tag{16}$$

and the calculated rank of $[T^1]$ is

$$\mathrm{Rank}(T^1)=10.$$

This $[T^1]$ sensitivity matrix is full rank due to the asymmetric ABC configuration.

Example 4: Using ABC System Sensitivities in Damage Detection

This example will demonstrate the use of a single ABC system (frequencies and sensitivity matrix) in place of the standard system frequencies and sensitivities. We continue to use the ABC system of Figs. 9 and 10, corresponding to Z-shaker excitation at DOF's 1 and 17. This ABC system yields rank =10 $[T^1]$ matrix, as shown above.

A 10% reduction in EI is made at element #3 and 15% reduction in El is made at element #4, yielding a simulated damaged structure. The natural frequencies of the undamaged (FE) and simulated damaged (Test) models are shown in Table 1.

Table 1. FE and Test Frequencies (KHz) and Error

Mode #	FE	Test	% Error
1	0.0313	0.0307	1.9
2	0.0863	0.0841	2.5
3	0.1693	0.1674	1.1
4	0.2803	0.2763	1.4
5	0.4198	0.4144	1.3
6	0.5887	0.5818	1.2
7	0.7882	0.7759	1.6
8	1.0180	1.0049	1.3
9	1.2646	1.2471	1.4
10	1.6794	1.6571	1.3

If the baseline system is used, i.e. $[T^0]$, the largest full rank submatrix of $[T^0]$ is 5 by 10, and its condition number

$$\mathrm{Cond}([T^0]) = 2.5e+02$$

The solution for the "damage" is shown in Figure 11, where the height of the bars corresponds to the magnitude of the element error. Given that the exact solution is 0.1 in element #3, and 0.15 in element #4, this solution provided by the baseline structure is poor.

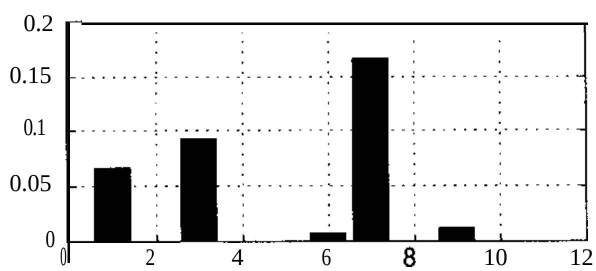

Figure 11. Baseline System Sensitvity Solution for Damage

If the ABC system sensitivities $[T^1]$ and mode frequencies (FE and experimentally identified) are used, one finds that the condition number is actually double, i.e.

$$\mathrm{Cond}([T^1]) = 5.1e+02$$

but the improved solution for the damage is shown in Fig. 12.

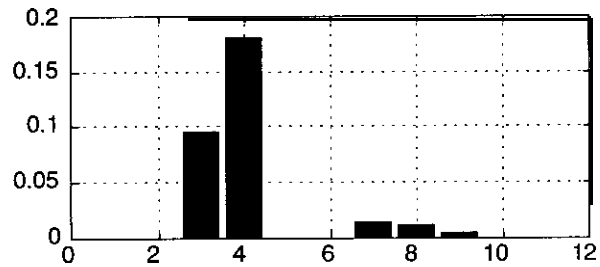

Figure 12. ABC System Sensitivity Solution for Damage

5. SUMMARY & DISCUSSION

It has been shown that the natural frequencies for a structure under a variety of boundary conditions are available from any square FRF matrix. The inversion of a spatially incomplete FRF matrix at each frequency in the test bandwidth yields impedance spectra which have peaks at the frequencies corresponding to the natural frequencies of structure restrained to ground at all the measured coordinates. The practical benefit of this result is that no physical alterations need be made to the structure, and no additional testing is required, hence the term "artificial boundary conditions (ABC)." The ABC configuration frequencies provide a larger database for model updating. For each ABC configuration, the FE model boundary conditions are altered in that "pins" are installed at the measured coordinates corresponding to the FRF matrix. This approach defines a single set of measured FRF's and several FE models, differing only in the boundary conditions.

With respect to the identification of the ABC frequencies (curvefitting), note that while the inverse of the measured FRF matrix defines and impedance matrix (Eq. (7)), near the ABC frequency the impedance spectra is dominated by the term Z^1_{oo}, and this suggests the use of standard single-degree-of-freedom curvefitting to identify the undamped ABC natural frequencies.

Given the baseline system frequencies, and additional ABC configuration frequencies, structural sensitivities can be generated from the baseline FE model, and from the model with the boundary conditions applied corresponding to the spatially incomplete FRF matrix. This expanded set of test data can be assembled into a linear system which can be solved in the context of either model updating or structural damage detection. It has been shown that the ABC configuration sensitivity matrix can be used in place of, or in addition to, the baseline configuration sensitivities, and can reduce or eliminate conditioning problems, such as those that arise when trying to discriminate damage in closely spaced elements.

6. ACKNOWLEDGMENTS

This work is dedicated to Hannah Marie Gordis, in honor of her first year.

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